

1. Find the  $y$ -intercept of the tangent line to the curve  $y = x^3$  at the point  $(2, 8)$ .

$$y' = 3x^2$$

$$f'(2) = 12$$

$$y - 8 = 12(x - 2)$$

$$y = 12x - 24 + 8$$

$$y = 12x - 16$$

- 2a Let  $f(x) = \sqrt{3x}$ . Find  $f'(3)$ .

$$f(x) = (3x)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(3x)^{-\frac{1}{2}} \cdot 3$$

$$f'(3) = \frac{1}{2}9^{-\frac{1}{2}} \cdot 3 = \left(\frac{1}{2}\right)$$

- 2b Given  $f(3) = 5$ ,  $f'(3) = 1.1$ ,  $g(3) = -4$  and  $g'(3) = 0.7$  find the value of  $(f \cdot g)'(3)$ .

$$(f \cdot g)'(3) = f'(3)g(3) + f(3)g'(3)$$

$$= 1.1(-4) + 5 \cdot 0.7$$

$$= -4.4 + 3.5 = -0.9$$

3. A spherical balloon is being inflated in such a fashion that its radius increases at a rate of 1 cm/s. In  $\text{cm}^3/\text{s}$ , how fast is the volume increasing 3 seconds after inflation starts?

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi(3)^2 1 \frac{\text{cc}}{\text{sec}}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$= 36\pi \frac{\text{cc}}{\text{sec}}$$

$$\frac{dr}{dt} = 1 \frac{\text{cm}}{\text{sec}} \Rightarrow r = 3 \text{ cm}$$

when  $t = 3 \text{ sec}$

4a. Find the value of the limit  $\lim_{x \rightarrow 0} \frac{\cos x - 1}{2x} = \lim_{x \rightarrow 0} \left( -\frac{1}{2} \cdot \frac{1-\cos x}{x} \right) = -\frac{1}{2} \cdot 0 = 0$

4b. Let  $f(x) = x \tan x$ . Find  $f'(\pi/4)$ .

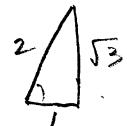
$$\begin{aligned} f'(x) &= (x)' \tan x + x (\tan x)' \\ &= \tan(x) + x \sec^2(x) \\ f'\left(\frac{\pi}{4}\right) &= \tan\left(\frac{\pi}{4}\right) + \frac{\pi}{4} \sec^2\left(\frac{\pi}{4}\right) \\ &= 1 + \frac{\pi}{4} \cdot 2 = \boxed{1 + \frac{\pi}{2}} \end{aligned}$$

5a. Let  $f(x) = (x+1)^4$ . Find  $f'(1)$ .

$$\begin{aligned} f'(x) &= 4(x+1)^3 \cdot 1 \\ f'(1) &= 4(1+1)^3 = \boxed{32} \end{aligned}$$

5b. Let  $f(x) = \sin^2(2x)$ . Find  $f'(\pi/6)$ .

$$\begin{aligned} f'(x) &= 2 \sin(2x) \cdot \cos(2x) \cdot 2 \\ f'\left(\frac{\pi}{6}\right) &= 4 \sin\left(\frac{\pi}{3}\right) \cos\left(\frac{\pi}{3}\right) \\ &= 4 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = \boxed{\sqrt{3}} \end{aligned}$$



5c. If  $f(x) = 4\sqrt{x} - \sqrt{x}$ , find the value of  $f'(4)$ .

$$\begin{aligned} f(x) &= 4\left(x - x^{\frac{1}{2}}\right)^{\frac{1}{2}} \\ f'(x) &= 4 \cdot \frac{1}{2} \left(x - x^{\frac{1}{2}}\right)^{-\frac{1}{2}} \left(1 - \frac{1}{2}x^{-\frac{1}{2}}\right) \\ f(4) &= 2(4 - \sqrt{4})^{-\frac{1}{2}} \left(1 - \frac{1}{2}4^{-\frac{1}{2}}\right) \\ &= 2 \cdot \frac{1}{\sqrt{2}} \left(1 - \frac{1}{4}\right) \\ &= \frac{2}{\sqrt{2}} \cdot \frac{3}{4} = \boxed{\frac{3}{2\sqrt{2}}} \end{aligned}$$

6a. If  $x^2 + xy + y^2 = 7$ , find the value of  $dy/dx$  at the point  $(1, 2)$ .

$$2x + y + xy' + 2yy' = 0$$

$$2(1) + 2 + (1)y' + 2(2)y' = 0$$

$$4 + 5y' = 0$$

$$y' = -\frac{4}{5}$$

6b. Find the slope of the tangent to the curve  $xy^2 + x^2y = 2$  at the point  $(1, 1)$ .

$$y^2 + x^2yy' + 2xy + x^2y' = 0$$

$$(1)^2 + (1)2(1)y' + 2(1)(1) + (1)^2y' = 0$$

$$3 + 3y' = 0$$

$$y' = -1$$

7. Let  $y = \frac{x}{(x+1)}$ . Find  $f''(0)$ .

$$y' = \frac{(x)'(x+1) - x(x+1)'}{(x+1)^2}$$

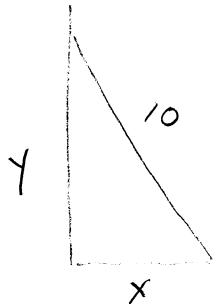
$$y' = \frac{x+1-x}{(x+1)^2} = (x+1)^{-2}$$

$$y'' = -2(x+1)^{-3} \cdot (1)$$

$$f''(0) = -2(0+1)^{-3} = \boxed{-2}$$

8.

A ladder 10 feet long is leaning against a wall, with the foot of the ladder 8 feet away from the wall. If the foot of the ladder is being pulled away from the wall at 3 feet per second, how fast in feet per second is the top of the ladder sliding down the wall?

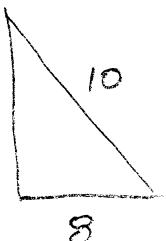


$$x^2 + y^2 = 10^2$$

$$\frac{dx}{dt} = 3 \frac{\text{ft}}{\text{sec}}$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

when  $x = 8$



$$\text{so } 2(8)(3) + 2(6) \frac{dy}{dt} = 0$$

$$y = \sqrt{10^2 - 8^2}$$

$$\approx 6$$

$$\frac{dy}{dt} = -\frac{2 \cdot 8 \cdot 3}{2 \cdot 6} = -\frac{4}{1} \frac{\text{ft}}{\text{sec}}$$